Problems 4

1. Let G be the group $< a > \times < b >$, where $|a| = 8,\ |b| = 12$. Let $K =< (a^2, b^3) >$. Compute the order of $\overline{(a^4, b)} \in G/K$.

2. Show that $\mathbb{Q}\,/\,\mathbb{Z}$ is an infinite abelian group in which every element has a finite order.

3. Let G be a group and H a subgroup. Recall that the *index* of H in G, denoted by $[G : H]$, is the number of left cosets of H. If there is a chain of inclusions of subgroups $K \subset H \subset G$, show that
$$[G : K] = [G : H][H : K].$$

4. Recall the circle group $\mathbb{C}^0 = \{z \in \mathbb{C}, |z| = 1\}$. Show that the group $\mathbb{C}^\times/\mathbb{C}^0$ is isomorphic to $(\mathbb{R}, +)$.

5. (From HW2). Classify the groups of order 6 by considering the following cases :
   (a) there is an element of order 6 ;
   (b) there is an element of order 3 and no element of order 6 ;
   (c) all elements have order 1 or 2.