

## Problems 5

1. Let  $G$  be a group. For any  $g \in G$ , consider the map  $\varphi_g : x \mapsto gxg^{-1}$ . We denote by

$$\text{Int}(G) := \{\varphi_g, g \in G\}.$$

- (a) Show that  $\text{Int}(G)$  is a normal subgroup of  $\text{Aut}(G)$ .
- (b) Consider the map  $f : G \rightarrow \text{Int}(G)$ ,  $g \mapsto \varphi_g$ . Show that  $f$  is a group homomorphism and compute its kernel.
- (c) Show that  $G/Z(G) \cong \text{Int}(G)$ .
2. Let  $G, H$  be groups of finite order such that  $|G| = m > 1$ ,  $|H| = n > 1$  with  $\gcd(m, n) = 1$ . Determine all the group homomorphisms  $f : G \rightarrow H$ .
3. Let  $G$  be a group. An element  $x \in G$  is called torsion element if  $\exists n > 0$ ,  $x^n = e$ . The group  $G$  is called *torsion group* if all its elements are torsion elements and *torsion free* if the only torsion element of  $G$  is  $e$ .
- Let  $G_1$  be a torsion group and  $G_2$  a torsion free group. Find all group homomorphisms  $G_1 \rightarrow G_2$ .