

Problems 7

1. Let $G = S_4$. Explain why G admits a subgroup of order 8 and exhibit one.
2. Same question for D_5 and orders 2 and 5.
3. Show that all reflections are conjugated in D_n .
4. Let p prime and $1 \leq n < p$. Let G be a group of order pn and H a subgroup of order p . Show that H is normal.
5. Recall that a group G is called *simple* if its only normal subgroups are $\{1\}$ and G itself. Let G be a group of order 200. Show that G cannot be simple.
6. Let G be a group of order 10.
 - (a) Show that there exists only one 5-Sylow $K = \langle x \rangle$, with x some element of order 5.
 - (b) Let H be a 2-Sylow. Explain why $H = \langle y \rangle$, with y some element of order 2 and show that $K \cap H = \{1\}$.
 - (c) Show that there exists $1 \leq r \leq 4$ such that $yx = x^r y$.
 - (d) Show that elements of the form $x^i y^j$ are distinct and deduce the group law of G .
 - (e) Show that r cannot be equal to 2, 3.
 - (f) Conclusion : show that $G \simeq C_5 \times C_2$ or $G \simeq D_5$.