NAME :

Quiz 2

1. Let G be a group. Define normal subgroup of G.
2. Consider the following subsets of $GL_n(\mathbb{R})$:

$$T = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}, ac \neq 0 \right\}, \quad N = \left\{ \begin{pmatrix} a & b \\ 0 & a^{-1} \end{pmatrix}, a \neq 0 \right\}.$$

   (a) Show that T is a subgroup of $GL_n(\mathbb{R})$.
   (b) Show that $\det : T \to \mathbb{R}^\times$ is an homomorphism of groups.
   ($\mathbb{R}^\times$ *is the group of non-zero real numbers with the usual multiplication ; no need to show that* $\mathbb{R}^\times$ *is a group*)
   (c) Show that N is a normal subgroup of T.
3. Show that $\phi : (\mathbb{Z}, +) \to (3\,\mathbb{Z}, +),\ n \mapsto 3n$ is an isomorphism of groups.